Reflector design

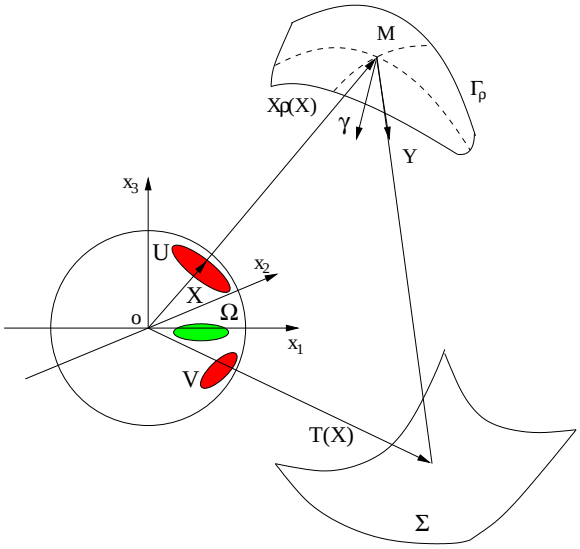
This problem concerns the existence and regularity of the surface M (see picture below) such that the light from o is reflected to the surface Σ. By computing the Jacobian of the mapping $T : X \in U \to T(X) \in \Sigma$, we get a Monge-Ampere type equation for ρ on the sphere, subject to the boundary condition $T(\partial U) = \partial\Sigma$.

The far field case. That is when Σ is at infinity. We consider instead the mapping $T : X \in U \to Y \in V$. In [rd1] we proved the existence, uniqueness, and (interior) regularity of solutions to the problem [rd1]. In [rd2] we proved that the reflector design is an optimal transportation. Therefore the linear programming can be used to obtain a numerical solution to the problem (except when M is rotationally symmetric, it was not at all clear how to get a satisfactory algorithm for a Monge-Ampere type equation with the special boundary condition $T(\partial U) = \partial V$).

Some ideas in [rd1] have been used in our work on the regularity of optimal transportation.

The near field case (the general case). In [rd3] we proved the existence and (interior) regularity of solutions to the problem. The near field case is much more complicated.

We would like to point out that when Σ lies in a plane passing through the origin, M satisfies the standard Monge-Ampere equation [rd3].



References

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[rd2] X.-J. Wang, On the design of a reflector antenna II, Calc. Var. PDE, 20(2004), 329-341.

[rd3] Aram Karakhanyan, X.-J. Wang, The reflector design problem, preprint.

[rd4] P. Guan, X.-J. Wang, On a Monge-Ampère equation arising in geometric optics, J. Differential Geometry, 48(1998), 205-223.