Remark. There are some very interesting websites available about mathematics, and careers in areas which use significant amounts of mathematics.

1) http://www.maths.anu.edu.au/MathematicsHandbook/ describes Math Department courses

2) At the top of the first page of the above there is a *link* to:
http://www.maths.anu.edu.au/MathematicsHandbook/whystudy.htm
which gives a few general ideas about mathematics in combination with other disciplines

3) The red arrow at the top of the prevous address is a *link* to
http://cs.jsu.edu/ms/ms-home.html
which is an excellent comprehensive US site which has links to just about everything on the Web that is mathematical (and not too flaky). Of particular interest is the careers heading, which gives some idea of the variety of fields which you can combine with mathematics.

4) An excellent site for career ideas is
http://www.ams.org/careers/

Misprint page 11 of the Calculus 1999 Notes line 9- should be

$$|a_n b_n - ab| \leq \epsilon,$$

not

$$|a_n b_n - ab| \leq ab.$$

AA1H, Assignment 5

You may talk to others about the problems. But you are expected to write out the solutions yourself, with no one else indicating what to write, and without help from anyone elses notes. If someone else had a major input into your solution, you should indicate this.

The assignment is due in by June 4, Friday 4pm (**NOT** 5pm as indicated in the general information sheet), in the AA1H collection box corresponding to your tutorial group, in the foyer of the Mathematics Department.

As usual, the ★ questions are for extra credit.

It looks like a long assignemnet, but it is not that bad. Questions 2–11 should not take more than 5 (say) lines each (Once you see how to do them!)

Exercise 1. Read Chapter 5 of the 1999 Calculus Notes.

Exercise 2. Let $f(x)$ be a continuous function and prove $|f(x)|$ is continuous.

Exercise 3. Where is the function $\ln(\sin x)$ defined and continuous?

Exercise 4. Suppose f is continuous on R and $f(q) = 0$ for all rationals q. Prove $f(x) = 0$ for all $x \in \mathbb{R}$.

Exercise 5. Suppose f is defined only on the integers. Explain why it is continuous.

Exercise 6. Find sequences (a_n) and (b_n) such that $a_n \to 0$ and $b_n \to 0$, but $\sin(1/a_n) \to 1$ and $\sin(1/b_n) \to 0$.

Exercise 7. Prove that $f(x) = x^3 - 4x + 2$ has a zero in the interval $[0, 1]$.

Exercise 8. Prove that all cubic polynomials have at leas one real root.

Exercise 9. Let f be a continuous function defined on a finite interval $[a, b]$. Suppose that $f(x) > 0$ for all $x \in [a, b]$. Prove that there is an $\alpha > 0$ such that $f(x) > \alpha$ for all $x \in [a, b]$.

Give a simple example to show this is not true if f is not continuous.

Exercise 10. Let f and g be continuous functions of $[a, b]$ and suppose $f(x) > g(x)$ for all $x \in [a, b]$. Prove that there is an $\alpha > 0$ such that $f(x) > \alpha + g(x)$ for all $x \in [a, b]$.

(You may use the previous result, even if you cannot prove it.)

Exercise 11. Suppose f is continuous on $[0, 1]$ and that $f(0) > 0$, $f(1) < 1$. Prove that $f(x) = x$ for some $x \in [0, 1]$.

Exercise 12. ★ Prove that at any time there are two antipodal points on the equator with the same temperature.

You may reformulate the problem as follows: Assume f is a continuous function on $[0, L]$ (think of L as the distance around the equator and f as the temperature). Assume $f(0) = L$. Prove there exists $x \in [0, L/2]$ such that $f(x) = f(x + L/2)$.

Exercise 13. ★ Assume that f is continuous at some point c. Prove that for every $\epsilon > 0$ there is a number $\delta > 0$ such that

$$|f(x) - f(c)| \leq \epsilon \quad \text{whenever} \quad |x - c| \leq \delta \text{ and } x \in \mathcal{D}(f).$$

HINT Assume for some $\epsilon > 0$ there is no such δ and obtain a contradiction.