AA1H, Assignment 3

You may talk to others about the problems. But you are expected to write out the solutions yourself, with no one else indicating what to write, and without help from anyone elses notes. If someone else had a major input into your solution, you should indicate this.

The assignment is due in by May 14, Friday 4pm (**NOT** 5pm as indicated in the general information sheet), in the AA1H collection box corresponding to your tutorial group, in the foyer of the Mathematics Department.

The ★ problem is for extra credit.

Exercise 1. Read the AA1H 1999 Calculus Notes from Remark 4.5 on page 5 to the end of 4.5 on page 13.

Exercise 2. Read Examples 4.13 and 4.14. Prove that the following sequences converge, and find their limits, by using Theorem 4.12.

1. $a_n = 5\left(1 + \frac{1}{\sqrt[3]{n}}\right)^2$.
2. $a_n = \frac{3n+1}{n+2}$.
3. $a_n = \frac{n^2+6}{3n^2-2}$.
4. $a_n = \frac{5+\left(\frac{2}{3^n}\right)^2}{2+\frac{2n+5}{3n-2}}$.

Exercise 3. Read Example 4.17. Prove that the following sequences converge, and find their limits, by using Theorem 4.16. You may use any of the standard properties of the sine, logarithm and exponential functions.

1. $a_n = e^{-n}\sin n$.
2. $a_n = (\sin n)\sin\frac{1}{n}$.
3. $a_n = (\cos n)(\ln n)^{-1}$ for $n \geq 2$.

Exercise 4.
Prove that the sequence in Exercise 7 of Assignment 2 converges to 4. That is, prove that if $a_{n+1} = \frac{1}{2}a_n + 2$, $a_1 = .5$, then $a_n \to 4$. *HINT*: subtract 4 from both sides.
Consider the sequence defined by the relation $a_{n+1} = \alpha a_n + 2$. Prove that if $|\alpha| < 1$ then the sequence has a limit independent of a_1.

What is the limit?

Exercise 5. ★ Let

$$f(x) = \lim_{n\to\infty}\left(\lim_{k\to\infty}\left(\cos(n!\pi x)\right)^{2k}\right).$$

Compute $f(x)$ for each real number x.

More misprints etc. from Notes

Thanks to those students pointing out the mistakes

1. Calculus 1999 Notes: Page 5 line 20; replace $N = 1+\left[\frac{\ln\frac{3}{\epsilon}}{\ln 2}\right]$ by $N = 2+\left[\frac{\ln\frac{3}{\epsilon}}{\ln 2}\right]$.
2. Calculus 1999 Notes: Page 5 bottom 4 lines; replace each of the 4 occurrences of L by a.
3. Calculus 1999 Notes: Page 6 line 22; replace $n \leq N$ by $n \geq N$.
4. Calculus 1999 Notes: Page 9 line 14; replace N_1 by N_2.
5. Calculus 1999 Notes: Page 10 line 17; replace $a_n \to 0$ by $a_n \to \frac{2}{3}$.
6. Algebra 1999 Notes: Page 18 line 20; replace "linearly independent" by "linearly dependent".