# HARMONIC ANALYSIS AND BOUNDARY VALUE PROBLEMS FOR ELLIPTIC SYSTEMS

This project concerns the interplay between operator theory, harmonic analysis and partial differential equations, with particular emphasis on boundary value problems and scattering theory of first order elliptic systems and related second order operators. Particular systems of interest are Stokes' equations and Maxwell's equations.

The operator theory developed by Kato was motivated principally by his study of evolution equations for parabolic and hyperbolic partial differential equations, as well as by related problems such as those arising in scattering theory. Although the work of the harmonic analysts has often been quite different, much of it, in particular that of Calderón, has been similarly motivated.

One specific aim is to study boundary value problems for linear partial differential equations with non-smooth coefficients on irregular domains. Results that are obtained under natural geometric conditions tend to be more useful when applied to those interesting nonlinear problems which arise from physical or geometric phenomena. They also have implications for parabolic and hyperbolic problems.

The principal methods involve developing the harmonic analysis of operators directly on domains or on their boundaries, building upon the great advances in harmonic analysis and operator theory made during the last twenty or thirty years by many people following on from Zygmund, Calderón, Stein and others. This involves proving square function estimates for the operators, and thus obtaining bounds on its functional calculus.

Particular topics for investigation are the following ones.

(i) The Kato square root problem for boundary value problems.

(ii) Quadratic estimates and functional calculi of perturbed Dirac operators.

(iii) Analyticity of spectral projections of Dirac operators on manfolds.

(iv) Sobolev spaces of forms on Lipschitz domains.

(v) Hardy spaces of exact forms on Riemannian manifolds.

(vi) Spectral theory and functional calculi in $L^p$ spaces.

Recent papers on some of these topics can be found at my website:
www.maths.anu.edu.au/~alan