ABSTRACTS OF TALKS FOR THE 8TH PACIFIC RIM GEOMETRY CONFERENCE

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Abstracts are listed alphabetically by author.

Iain Aitchison *(University of Melbourne)*
Crystallographic Phyllotaxy: Cosmic origins of compressed diamond
Abstract: (Joint with Ryuji Abe) The "Da Vinci Code" notwithstanding, the Golden Ratio and Fibonacci numbers play a significant role in number theory and geometry. We describe an analogue of the relationship between Diophantine approximation by rationals, Markov and Lagrange spectra, and geodesics on hyperbolic punctured tori in one dimension higher. More precisely, the approximation of complex numbers by Gaussian rationals leads to an analysis of geodesics in the Borromean ring complement, with the surprising emergence of the diamond lattice in 3-dimensional Euclidean space carrying a class of geodesics related to number theoretic spectra of poorly approximable numbers. The diamond lattice itself arises from integer lattice points in Minkowski space.

Maria Athanassenas *(Monash University)*
Boundary gradient estimates for prescribed mean curvature surfaces
Abstract: We discuss a short, geometric approach to proving gradient estimates on the boundary for surfaces of prescribed mean curvature, under appropriate conditions. The method applies in particular to capillary surfaces in a tube, with Neumann boundary data.

Simon Brendle *(Stanford University)*
Deformation of Riemannian metrics by their scalar curvature and related problems
Abstract: We will discuss recent results regarding two questions that arise in connection with the Yamabe problem. The first problem is concerned with the conformal deformation of Riemannian metrics by their scalar curvature. We are interested in the asymptotic behavior of the flow as $t \to \infty$. The second problem is concerned with the set of constant scalar curvature metrics in a given conformal class. This set is known to be compact for most background metrics, but blow-up is possible for certain background metrics in high dimensions.

Maria-Cristina Caputo *(University of Texas at Austin)*
Highly Degenerate Harmonic Mean Curvature Flow
Abstract: We study the evolution of a weakly convex surface in $\mathbb{R}^3$ with flat sides by the Harmonic Mean Curvature Flow. We establish the short time existence as well as the optimal regularity of the surface and show that the boundaries of the flat sides evolve by the Curve Shortening Flow. It follows from our results that a weakly convex surface with flat sides of class $C^{k,\gamma}$, for some $k \geq 1$, $0 < \gamma \leq 1$ remains in the same class.

Jingyi Chen *(University of British Columbia)*
Minimal cones with isotropic links
Abstract: We show that any closed oriented immersed isotropic minimal surface Σ with genus g_Σ in $S^5 \subset \mathbb{C}^3$ is (1) Legendrian (and totally geodesic) if $g_\Sigma = 0$; (2) either Legendrian or with exactly $2g_\Sigma - 2$ Legendrian points if $g_\Sigma \geq 1$. In general, any compact oriented immersed isotropic minimal submanifold $L^{n-1} \subset S^{2n-1} \subset \mathbb{C}^n$ must be Legendrian if its first Betti number is zero. Corresponding results for non-orientable links are also provided.

Philippe Delanoe *(Université de Nice)*
Kazdan-Warner identities versus absence of constraints for Nirenberg type problems
Abstract: The Nirenberg problem consists in characterizing the functions on the n-sphere which are Gauss ($n = 2$) or scalar ($n \geq 3$) curvature of riemannian metrics pointwise conformal to the standard one g_0. If f is such a function for the conformal metric $g_u = e^{2u} g_0$, Kazdan and Warner (1974-75) observed that necessary constraints bear on the couple (f, u). Lately, it was proved that no constraint can ever bear solely on f. Here, we will explain the latter result and show how both results extend to other conformal curvature functions, like the so-called σ_k-curvatures and the Q-curvatures (the latter was proved in collaboration with F. Robert at Nice).

Michael Eastwood *(University of Adelaide)*
Zero energy fields on complex projective space
Abstract: I shall indicate how various geometric tricks can be used to study real integral geometry (variants of the Radon transform) on complex projective space viewed as a Riemannian manifold with its usual Fubini-Study metric. This is joint work with Hubert Goldschmidt.

A Rod Gover *(University of Auckland)*
Prolonged differential systems and a holonomy characterisation of the Fefferman Space
Abstract: It is a result of Charles Fefferman that to each CR manifold (of hypersurface type and non-degenerate) one can canonically associate a conformal manifold. While there have been many applications of this structure it would seem that most of its potential remains untapped. In part this is is due to the complicated relationship between the obvious local CR geometric objects and their conformal counterparts. Appropriate prolonged systems yield a vastly simpler picture. This is illustrated by a conformal characterisation of the Fefferman space as simply a certain holonomy reduction of the conformal tractor bundle. We also shed light on whether a Fefferman space may admit an Einstein metric: It was stated by J. Lee that "the Fefferman metric is *never* Einstein". We strengthen this result while at the same time showing that (following Leitner), upon restriction to suitable open dense sets, the Fefferman conformal class may in fact admit Einstein metrics. This is joint work with Andreas Cap.

Joseph Grotowski *(University of Queensland)*
How to blow hyperbubbles
Abstract: We examine the differences, both topological and analytical, between weak and strong limits for minimizing sequences for the higher-dimensional analogue of the Dirichlet energy.

Kengo Hirachi *(University of Tokyo)*
The ambient metric to all orders in even dimensions
Abstract: The ambient metric associated to a conformal manifold is an important object in conformal geometry. However, the basic construction is obstructed at finite order in even dimensions. This talk will describe how to complete the construction to all orders in even dimensions. One obtains a family of smooth ambient metrics determined up to smooth diffeomorphism. These ambient metrics arise as an invariantly defined smooth part of inhomogeneous Ricci-flat metrics with asymptotic expansions involving log terms. This is joint work with Robin Graham.

Min-Chun Hong *(University of Queensland)*
On the ASD connection flow in dimension 4
Abstract: I will discuss some new results about the ASD connections and their flow in principal bundles over a 4-dimensional closed and oriented Riemannian manifold.

Kang-Tae Kim *(Pohang University of Science and Technology)*
On the Omori-Yau Generalized Maximum Principle
Abstract: Well-known Omori-Yau Generalized Maximum principle has been generalized by several excellent authors. In this talk, we will present a version of proof. This may (we hope) be useful toward a study of some problems concerning the Bergman metric and the behavior of its Ricci tensor. This is from a collaboration between Hanjin Lee and the speaker.

Finnur Larusson *(University of Adelaide)*
Polynomial hulls and analytic discs
Abstract: The polynomial hull $\hat{K}$ of a compact set K in complex affine space is the set of all points z such that $|P(z)| \leq \sup_K |P|$ for all complex polynomials P. If z lies in an analytic disc with boundary in K, then it easily follows from the maximum principle that $z \in \hat{K}$. A famous example of Stolzenberg (1963) shows that the converse fails. The question of whether polynomial hulls could still be somehow described in terms of analytic discs remained open until 1993, when Poletsky gave a description using his theory of disc functionals. Earlier this year, Ragnar Sigurdsson and I obtained a different description as an application of our work, published last year, on the so-called Siciak-Zahariuta extremal function. The gist of both results is to suitably relax the boundary conditions on the analytic discs. I will describe my work with Sigurdsson and its background.

Gaven Martin *(Massey University)*
Curvature and dynamics
Abstract: We discuss higher dimensional analogues of the classical Fatou-Julia theory of conformal dynamics of the 2-sphere. There are generally quite strong rigidity phenomena for smooth conformal mappings, but there are examples on higher dimensional Spheres and Lens spaces of maps conformal and non-injective with respect to some measurable Riemannian structure. The generalised Lichnerowiz problem asks us to classify those manifolds which admit a rational map of degree two or more. A generalisation of a result of Sela on word hyperbolic groups (the "virtual Hopf property") together with results of Walsh and Smale give powerful tools for analysing the dynamics of non-injective open maps of negatively curved spaces - this in turn gives strong generalisations of Mostow rigidity and so forth. Recent applications to the the structure of the singular set of an open mapping and open maps of knot complements are also found. This is joint work with M. Bridson D. Groves and J. Hillman.

Pengzi Miao *(Monash University)*
On an inequality between ADM mass and boundary capacity
Abstract: To motivate the content of this talk, we start by recalling a theorem of Bunting and Masood-ul-Alam (1987) on the uniqueness of static black hole spacetime. Then we relate it to a result of Bray (2001) on an inequality between ADM mass and boundary capacity when the boundary consists of minimal surfaces. Finally, we provide a similar inequality which generalizes Bray's result in a special case to allow arbitrary boundary. This is a joint work with Hugh Bray.

Yuri Nikolayevsky *(Latrobe University)*
Einstein solvmanifolds and their nilradicals
Abstract: There are two main open questions in the theory of noncompact homogeneous Einstein spaces. The first one is the Alekseevsky Conjecture: any Einstein noncompact homogeneous space is a solvmanifold (a solvable Lie group with a left-invariant Riemannian metric). The second one, due to Heber, arises from the fact that all the known examples of Einstein solvmanifolds are standard (which means that the orthogonal complement to the nilradical of the corresponding Lie algebra is abelian). Are there any nonstandard Einstein solvmanifolds? We give a partial negative answer to that question. It is shown that many classes of nilpotent Lie algebras (abelian, some two -step nilpotent, filiforms, free Lie algebras, nilpotent algebras of dimension at most 7, Einstein nilradicals) do not contain the nilradicals of nonstandard Einstein solvmanifolds. We also prove that there are no nonstandard Einstein solvmanifolds of dimension less than ten.

Takeo Ohsawa *(Nagoya University)*
Hartogs type continuation and its application to CR geometry
Abstract: For real analytic hypersurfaces in complex manifolds, Levi flatmess is equivalent to local existence of pluriharmonic defining functions. Accordingly, Levi flat hypersurfaces are naturally related to some global existence questions in complex analysis. It has been known that there exist Stein domains of dimension two that bound Levi flat hypersurfaces. Recently it turned out that a compact Kaehler manifold of dimension at least 3 does not contain a real analytic Levi flat hypersurface whose complement is Stein (to appear in Nagoya Math. J.). The purpose of my talk is to explain that this result is a consequence of Hartogs type continuation theorems for the cohomology classes, which were obtained long time ago by Grauert-Riemenschneider and by myself. Related results for divisors with Stein complements will be discussed, too.

Tim Riley *(Cornell University)*
The geometry of discs filling loops in groups and manifolds
Abstract: There are two natural ways to measure the diameter of a combinatorial disc D (a van Kampen diagram) spanning a loop the Cayley 2-complex of a finitely presented group: either intrinsically, using the path metric on the 1-skeleton of D, or extrinsically, using the path metric on the Cayley graph. One expects intrinsic diameter to be qualitatively larger than extrinsic diameter, in general. I will give groups that confirm this intuition and will translate the construction to the setting of discs spanning loops in closed Riemannian manifolds. The techniques involved also speak to issues concerning filling length - an invariant concerning the length of the contracting curve in the course of a null-homotopy of a loop. Time allowing, I will show that filling length varies dramatically depending on whether or not one considers free or basepoint-fixed null-homotopies. This is joint work with Martin Bridson.

J. Hyam Rubinstein *(University of Melbourne)*
Problems around 3-manifolds
Abstract: With the spectacular solution to Thurston's geometrisation conjecture, there are still interesting questions remaining about 3-manifolds and related topics. I will discuss a number of such problems, organised into groups around the themes of minimal surfaces, Ricci flow, combinatorial differential geometry, 4-manifolds which behave like 3-manifolds, contact structures and Heegaard splittings and singular incompressible surfaces.

Gerd Schmalz *(University of New England)*
Multicontact structures
Abstract: I will discuss a geometric structure that is defined by a set of distinguished direction fields. It will be shown that in the case of $2n$ direction fields that span a contact distribution in $2n+1$ dimensional space the algebra of infinitesimal symmetries is of finite dimension not exceeding $3n+2$ (when $n > 1$). The case $n = 1$ is equivalent to the geometry of point-transformations for 2nd order ODE and has been studied by Sophus Lie in 19th century. For $n > 1$ the structure becomes more rigid but irregular in the sense of Tanaka's theory. We will show that the obtained estimate for the symmetry algebra is sharp. This is joint work with C.-K. Han and J.-W. Oh (Seoul National University).

Mathai Varghese *(University of Adelaide)*
Equivariant and fractional index of projective elliptic operators
Abstract: I will report on joint work with R. B. Melrose and I. M. Singer `[math.DG/0611819]`

Bryan Wang *(Australian National University)*
Geometry of 3-dimensional monopoles
Abstract: I will briefly present geometrcal and analytical aspects of the Seiberg-Witten monopoles on 3-dimensional manifold with/without boundary. Some applications and open problems will be discussed.

Norman Wildberger *(University of New South Wales)*
Chromogeneity
Abstract: Universal geometry develops metrical geometry in a purely algebraic fashion, valid over a general field and in fact an arbitrary quadratic form, by replacing distance and angle with the quadratic concepts of quadrance and spread. The main algebraic laws governing the situation are the five main laws of rational trigonometry, as developed in the recent book 'Divine Proportions: Rational Trigonometry to Universal Geometry'. Chromogeometry arises from universal geometry, and studies a remarkable three fold symmetry in planar geometry–the usual Euclidean theory is one third of a bigger picture involving two relativistic geometries, and all three interact in a way which transcends Klein's Erlangen program! The talk will introduce you to this new way of thinking, mainly through lots of (colour) pictures, involving Euler lines of triangles, foci and directices of parabolas and other conic sections, centers of quadrilaterals and more.

Weiping Zhang *(Chern Institute of Mathematics, Nankai University)*
Cheeger-Mueller theorem for complex valued Ray-Singer torsion
Abstract: We report a joint work with Guangxiang Su, in which we generalize a theorem of Bismut-Zhang, which extends the Cheeger-Mueller theorem on Ray-Singer torsion and Reidemeister torsion, to the case where the flat vector bundle over a closed manifold carries a nondegenerate symmetric bilinear form. As a consequence, we prove the Burghelea-Haller conjecture which gives an analytic interpretation of (the square of) the Turaev torsion.