The AMSI-ANU Workshop on Spectral Theory and Harmonic Analysis

The Mathematical Sciences Institute
The Australian National University, Canberra
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Sponsors
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	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
9.20	Registration and Morning Tea	Expository Lecture *Smith*	Expository Lecture *Smith*	Expository Lecture *van Neerven*	Lecture *ter Elst*
10.20	Welcome Venue: The Tank	Morning Tea			
10.50	Expository Lecture *Smith* (starting 10.40)	Lecture *Guillarmou*	Lecture *Sogge*	Lecture *Rios*	Lecture *Nonnenmacher*
11.40	Lecture *Bouclet*	Lecture *Alexandrova*	Lecture *Blair*	Lecture *Yan*	Lecture *Hillaret*
12.30	Lunch				
1.50	Expository Lecture *van Neerven*	Expository Lecture *van Neerven*		Lecture *Datchev*	Lecture *Doust*
2.50	Lecture *Hytönen*	Lecture *Toro*		Lecture *Sikora* (starting 2.40)	Lecture *M. Mitrea*
3.40	Afternoon Tea			Afternoon Tea (starting 3.30)	
4.10	Lecture *Rule*	Lecture *Martell*		Colloquium *Albin* (starting 4pm)	
5.00	Reception (until 6.30pm)	Workshop Dinner (6.30 for 7pm)		Poster Session (until 6.30pm)	

List of Venues

Monday morning registration:
MSI Common Room (Room 1175), John Dedman Building (Building 27)

Morning and afternoon teas:
MSI Common Room (Room 1175), John Dedman Building (Building 27)

Welcome:
The Haydon–Allen Lecture Theatre (Building 23, also known as 'The Tank')

Lectures:
The Haydon–Allen Lecture Theatre (Building 23, also known as 'The Tank')

Monday evening reception:
MSI Common Room (Room 1175), John Dedman Building (Building 27)

Thursday colloquium:
The Haydon–Allen Lecture Theatre (Building 23, also known as 'The Tank')

Poster session:
Foyer of Manning Clark Centre (Building 26a, between John Dedman Building and Union Court)

Workshop dinner:
Teatro Vivaldi Restaurant (Arts Centre, Building 16, adjacent to Union Court)

Workshop Proceedings

The organisers are planning to publish proceedings of the workshop in the *Proceedings of the Centre for Mathematics and its Applications*. This will be a refereed publication. Papers should normally be between 5 and 30 pages long, and on some topic in spectral theory and harmonic analysis. Survey articles as well as announcements of new results are welcome. Submissions are due on 1 October 2009. For further information on submission, please visit the workshop website at

http://wwwmaths.anu.edu.au/events/SpectralTheory09/

Abstracts for Expository Lectures

Stochastic evolution equations in UMD Banach spaces
Jan van Neerven, Delft University of Technology
Monday 1.50, Tuesday 1.50, Thursday 9.20

Abstract: Much of the recent progress in both vector-valued harmonic analysis and vector-valued stochastic analysis is based on the idea that vector-valued extensions of classical Hilbert space-valued results can be obtained if one replaces:

- Hilbert spaces H by *UMD Banach spaces* X,
- H-valued L^2-functions by γ*-radonifying operators* from L^2 into X,
- Uniformly bounded $\mathscr{L}(H)$-valued functions by γ*-bounded* $\mathscr{L}(X)$*-valued functions*.

Here the prefix 'γ-' stands for 'Gaussian'.

In these lectures we show how this paradigm is used in stochastic analysis to study stochastic evolution equations of parabolic type. We begin by explaining how PDEs driven by space-time white noise can be reformulated as abstract stochastic stochastic evolution equations. In order to solve such equations one needs an extension of the classical Itô stochastic calculus to the Banach space-valued setting. As it turns out, for UMD Banach spaces X this can be done in a very satisfactory way using the above circle of ideas. Roughly speaking, the natural class of X-valued processes for which an analogue of the Itô isometry can be proved are those processes whose trajectories define γ-radonifying operators. γ-Bounded families of operators act as multipliers on such processes. We show how this theory can be applied to stochastic PDEs to prove existence, uniqueness, and space-time regularity of solutions.

Much of the recent work presented here is joint work with Mark Veraar (Delft) and Lutz Weis (Karlsruhe).

Wave Packet Methods for Hyperbolic Equations

Hart Smith, University of Washington

Monday 10.40, Tuesday 9.20, Wednesday 9.20

Abstract: In this series of expository talks, I will present the wave packet transform, and discuss its usefulness for representing solutions to hyperbolic (wave) equations, and for establishing L^p bounds (Strichartz estimates) on those solutions.

Wave packets methods were originally used with great success to study the behaviour of solutions to variable coefficient wave equations where the wave-speed coefficients are of limited differentiability. More recently, they have been adapted to study solutions of the wave equation on manifolds with boundary (with e.g. Dirichlet conditions imposed), and have allowed one to establish Strichartz bounds in this setting as well.

Abstracts from Invited Speakers

Ricci flow and the determinant of the Laplacian on non-compact surfaces
Pierre Albin, Massachusetts Institute of Technology
Thursday 4.00 (Colloquium)

Abstract: To extend the determinant of the Laplacian to non-compact surfaces one has to deal with the presence of continuous spectrum and the fact that the heat kernel is not trace-class. I will explain how to use renormalized integrals to extend the definition of the determinant and find a formula for its variation among asymptotically regular metrics. I will also report on the behaviour of normalized Ricci flow on these metrics and use this to show that the maximum value of the determinant of the Laplacian occurs at constant curvature metrics. This is joint work with Clara Aldana and Frederic Rochon.

Infinite Semi-Classical Fourier Integral Operators and Beals's Lemma
Ivana Alexandrova, East Carolina University / University of Tokyo
Tuesday 11.40

Abstract: I discuss a new class of semi-classical Fourier integral operators and show that this is the right class for the semi-classical interpretation of Beals's lemma.

Strichartz Estimates for the Schrödinger Equation in Exterior Domains
Matthew Blair, University of New Mexico
Wednesday 11.40

Abstract: Strichartz inequalities are a family of space-time integrability estimates for the Schrödinger equation that rely on the dispersive effect of the solution map. These estimates are reasonably well-understood when the equation is posed over Euclidean space. However, the situation is more complicated when one starts to consider the obstacle problem, as the local and global geometry of the boundary can influence how solutions develop. We will discuss recent progress in proving estimates for these boundary value problems, with emphasis on a joint work with Smith and Sogge. Applications to semilinear Schrödinger equations will also be highlighted.

Propagation estimates for the Schrödinger equation
Jean-Marc Bouclet, Laboratoire Paul Painlevé, University of Lille 1
Monday 11.40

Abstract: We shall describe recent results on low frequency estimates for derivatives of the resolvent of asymptotically Euclidean Laplacians, and related time decay estimates.

Soliton evolution in the Hartree equation with slowly varying potential
Kiril Datchev, UC Berkeley
Thursday 1.50

Abstract: We study the Hartree equation with a slowly varying smooth potential, $V(x) = W(hx)$, and with an initial condition which is ϵ away in H^1 norm from a soliton. We show that up to time $\log(1/h)/h$ and errors of size $\epsilon + h^2$ in H^1, the solution is a soliton evolving according to the classical dynamics of a natural effective Hamiltonian, which we compute explicitly. This result is based on methods of Holmer-Zworski, who prove a similar theorem for the Gross-Pitaevskii equation, and on spectral estimates for the linearized Hartree operator recently obtained by Lenzmann.

A maximal theorem for holomorphic semigroups
Ian Doust, University of New South Wales
Friday 1.50

Abstract: Let X be a closed linear subspace of the Lebesgue space L^p for some $1 < p < \infty$, and let A be an invertible operator that is the generator of a bounded holomorphic semigroup T_t on X. Then for each $0 < \alpha < 1$ the maximal function $\sup_{t>0} |T_t f(x)|$ belongs to L^p for each f in the domain of A^α. If moreover iA generates a bounded C_0?group and A has spectrum contained in $(0, \infty)$, then A has a bounded H^∞ functional calculus. This is joint work with Gordon Blower (Lancaster).

Does diffusion determine the manifold?
Tom ter Elst, University of Auckland
Friday 9.20

Abstract: The famous question of Kac is whether one can hear the shape of a drum. Or more precisely, whether all eigen frequencies of a drum determine the drum. In general the answer to the latter question is negative. The eigen frequencies are equal if and only if there exists a unitary operator which maps the Laplacian on the first drum onto the Laplacian on the second drum. In this talk we discuss what happens if the unitary operator is replaced by an order isomorphism, i.e., if it maps positive functions to positive functions. Or equivalently, if the diffusion processes on the two drums are equal.

This is joint work with M. Biegert and W. Arendt.

Calderon inverse problem on Riemann surfaces
Colin Guillarmou, University of Nice
Tuesday 10.50

Abstract: We address the problem of the determination of an electric potential or conductivity on a Riemann surface with boundary from the Dirichlet-to-Neumann operator acting on functions supported in an open set of the boundary. This is joint work with Leo Tzou (Stanford).

Generic simplicity of triangles
Luc Hillairet, Laboratoire de Mathématiques Jean Leray
Friday 11.40

Abstract: We will show that the generic Euclidean triangle has simple Laplace spectrum.

L^p theory of Kato's square roots and more
Tuomas Hytönen, University of Helsinki
Monday 2.50

Abstract: In a joint work with A. McIntosh and P. Portal, we have extended parts (in particular, the harmonic-analytic ones) of the solution of Kato's square root problem in L^2 to other L^p spaces (JFA, 2008). More recently, we have been dealing with the related holomorphic calculus questions in the broader framework of variable-coefficient Hodge-Dirac operators, where new operator-theoretic challenges had to be overcome especially in connection with possibly non-trivial null spaces. When everything is set up and proven, the consequences include quite general perturbation results for the operators under investigation.

A_∞ estimates via extrapolation of Carleson measures
José María Martell, Consejo Superior de Investigaciones Cientificas
Tuesday 4.10

Abstract: In joint work with S. Hofmann, we revisit the "extrapolation" method for Carleson measures, originally introduced by John Lewis to prove A_∞ estimates for certain caloric measures. We present a purely real variable version of the method suitable for deducing that a weight is in A_∞, given appropriate control by a Carleson measure. To illustrate the applicability of this criterion, we reprove a well known theorem of R. Fefferman, Kenig and Pipher concerning the solvability of the Dirichlet problem with data in some L^p space. We also adapt a result of Dahlberg, Jerison and Kenig to our dyadic setting in order to compare the harmonic measures on discrete sawtooth domains.

Geometric analysis and PDEs: the role of boundary oscillations
Marius Mitrea, University of Missouri
Friday 2.50

Abstract: I will discuss recent progress in the area interfacing geometric analysis, geometric measure theory, harmonic analysis and elliptic boundary value problems, with special emphasis on the role of the infinitesimal mean oscillations of the coefficients of the operator and/or the unit normal for the underlying domain. Specific cases include higher-order systems of PDEs with bounded, measurable coefficients in Lipschitz domains, and constant coefficient PDEs in NTA domains.

Eigen- and quasi-modes of Anosov manifolds
Stéphane Nonnenmacher, Institut de Physique Théorique, CEA
Friday 10.50

Abstract: We want to describe the localization properties of high-frequency eigenmodes of the Laplacian on compact Riemannian manifolds with negative curvature. For that aim, we consider the semiclassical measures associated with (infinite) sequences of eigenmodes, and show that the metric entropy of these measures is necessarily bounded from below by a positive number, equal to half the maximal entropy in the case of constant negative curvature. The same method allows to control the localization properties of quasimodes of order $C\lambda/\log\lambda$, and shows a "localization crossover" between small and large values of C. Collaboration with Nalini Anantharaman.

The Square Root Problem for A2-Elliptic Operators with Gaussian Bounds
Cristian Rios, University of Calgary
Thursday 10.50

Abstract: We extend the positive answer to the famous Kato square root problem under Gaussian bounds assumptions given by Hofmann, Lacey, and McIntosh (2002). We consider divergence form operators with ellipticity controlled by a Muckemphout A2 weight. We show that if the associated heat kernel satisfies Gaussian upper bounds then the square root operator is equivalent to the gradient in the weighted L^2 space. This work is done in collaboration with David Cruz-Uribe.

Weighted norm inequalities for pseudo-differential operators
David Rule, University of Edinburgh
Monday 4.10

Abstract: We prove weighted norm inequalities for pseudo-differential operators whose symbols are only measurable in the spatial variable, namely for symbols in the class $L^\infty S^m_\rho$ introduced by Kenig and Staubach with the same requirements on m and ρ. We obtain these results by proving appropriate maximal function inequalities. We can improve the range of m if we restrict ourselves to symbols of a particular form, which are still only required to be measurable in the spatial variable. As an application of these weighted boundedness results we prove that the commutator of such operators with a function of bounded mean oscillation is a bounded operator on (unweighted) L^p. This is joint work with Nick Michalowski and Wolfgang Staubach.

Bochner–Riesz summability and restriction theorems on asymptotically conic manifolds
Adam Sikora, Macquarie University
Thursday 2.40

Abstract: This presentation is based on a collaborative research project with Colin Guillarmou and Andrew Hassell.

The Bochner-Riesz analysis and restriction theorems are classical problems in harmonic analysis. Such results are motivated by the fundamental question of convergence of Fourier series and the inverse Fourier transform. These theorems can be interpreted as statements about the spectral projection of the standard Laplace operator. We study the Bochner-Riesz summability and restriction theorems in the setting of spectral analysis of the Laplace operators on a certain class of curved Riemannian manifolds.

Abstract Strichartz estimates and existence theorems for nonlinear wave equations

Christopher Sogge, Johns Hopkins University
Wednesday 10.50

Abstract: I shall go over recent work on proving "abstract Strichartz estimates" that can be used to solve small-amplitude wave equations of the form $\Box_g u = |u|^p$ for a sharp range of exponents p. These results hold for obstacle problems and for certain noncompact perturbations g of the standard metric g_0 in Minkowski space.

Analysis on non-smooth domains

Tatiana Toro, University of Washington
Tuesday 2.50

Abstract: The theory of "tent spaces" developed by Coifman, Meyer and Stein has played a crucial role in studying questions concerning the Dirichlet problem for elliptic equations on Lipschitz domains. In this talk we will show that "tent spaces" can be defined and behave well on chord-arc domains. This yields results concerning the solvability of the Dirichlet problem for elliptic equations on this type of domains. This is joint work with E. Milakis and J. Pipher.

Hardy spaces and regularity of the second order derivatives for the Dirichlet and Neumann problems on bounded convex domains

Lixin Yan, Macquarie University/Sun Yat-sen University
Thursday 11.40

Abstract: In this talk we will describe some Hardy spaces associated to the Dirichlet and Neumann Laplacian on a bounded Lipschitz domain in $\mathbb{R}^n$. We then show regularity of the Green's operators of the Dirichlet and Neumann problems in the context of the Hardy spaces associated with these operators on a bounded convex domain Ω of $\mathbb{R}^n$. This is a joint work with X.T. Duong, S. Hofmann, D. Mitrea and M. Mitrea.

Abstracts for Poster Session

Homogeneous right inverses for operators on anisotropic Besov, Lizorkin–Triebel and other quasi-Banach spaces

Sergey Ajiev, University of New South Wales

Thursday 5.00 (Poster Session)

Abstract: Some problems, for example, in PDE can be reduced to the invertibility of a closed operator with non-trivial and non-complemented kernel defined, for example, on a Sobolev space. We construct homogeneous (non-linear) right-inverse operators and establish explicit estimates for the exponents of their Hölder regularity and the corresponding Hölder seminorms in both abstract and particular settings. In particular, we deal with various types of Besov and Lizorkin–Triebel spaces of functions on an arbitrary open subset of an Euclidean space defined in terms of local approximations, differences, or a functional calculus and also with their duals, subspaces, quotients and a wide class of "independently generated spaces."

Absolutely continuous and singular spectral shift functions

Nurulla Azamov, Flinders University

Thursday 5.00 (Poster Session)

Abstract: A natural decomposition of the spectral shift function into absolutely continuous and singular parts is discussed. Under certain conditions on the self-adjoint operator and its perturbation, in the Birman-Krein formula the spectral shift function can be replaced by its absolutely continuous part. This implies that the singular spectral shift function is almost everywhere integer-valued.

This suggests that the singular part of SSF measures the spectral flow of the singular spectrum, in particular, it measures the spectral flow of eigenvalues embedded in continuous spectrum. I conjecture that the singular part of SSF is a.e. integer-valued for trace compatible perturbations of a self-adjoint operator.

The Klein–Gordon equation on asymptotically de Sitter spaces
Dean Baskin, Stanford University
Thursday 5.00 (Poster Session)

Abstract: The de Sitter space is a spherically symmetric solution of the vacuum Einstein equations with a positive cosmological constant. We consider the wave and Klein-Gordon equations on asymptotically de Sitter spaces with no caustics. We construct a parametrix for the fundamental solution of the Klein-Gordon equation. We use the parametrix to establish asymptotic expansions for solutions of the Klein-Gordon equations, a family of L^p estimates, and a family of Strichartz estimates for the linear equation.

Recent progress on isolated singularities for elliptic equations via regular variation theory
Florica Cîrstea, The University of Sydney
Thursday 5.00 (Poster Session)

Abstract: Initiated by Karamata in 1930s, the regular variation theory was later extended and developed by many others, playing an important role in certain areas of probability theory such as in the theory of domains of attractions and max-stable distributions. Despite having found beautiful applications in many fields, Karamata's theory had been only sporadically used in partial differential equations. We will illustrate how regular variation theory and PDEs can be intertwined leading to new approaches and further progress on the much-studied topic of isolated singularities for nonlinear elliptic equations.

An isoperimetric inequality for the first eigenvalue of Robin problems
Daniel Daners, The University of Sydney
Thursday 5.00 (Poster Session)

Abstract: Consider the first eigenvalue of the Laplace operator on a bounded domain with Dirichlet boundary conditions. The well known Faber-Krahn inequality asserts that, amongst all domains of equal volume, the ball minimizes the first eigenvalue. The corresponding conjecture for Robin boundary conditions has only been resolved recently. The main reason is that symmetrisation techniques do not seem to work if the eigenfunction is not constant on the boundary of the domain. We present an alternative proof for the p-Laplace operator with Robin boundary conditions. This is joint work with James Kennedy and Dorin Bucur.

Lie Symmetries for some Hyperbolic PDEs
Stephen Godfrey, University of Technology, Sydney
Thursday 5.00 (Poster Session)

Abstract: Here we discuss a class of solutions of a more general axisymmetric wave equation of the form $u_{tt} = u_{xx} + f(x)u_x$, using symmetry methods. We show that there are non-trivial symmetries if and only if f is a solution of one of a family of Riccati equations. We study the symmetries and show how they may be used to construct fundamental solutions.

Detecting Einstein geodesics
A. Rod Gover, University of Auckland
Thursday 5.00 (Poster Session)

Abstract: We discuss links between projective and conformal geometry and the applications of these to the following problem: Given a torsion-free connection ∇ on $M^{n\geq 3}$, do its geodesics, as unparametrised curves, coincide with the geodesics of an Einstein metric? This is joint work with Heather Macbeth.

On generalised Wentzell boundary conditions
James Kennedy, University of Sydney
Thursday 5.00 (Poster Session)

Abstract: We give an overview of the theory of second order elliptic equations $Au = f$ in some domain $\Omega \subset \mathbb{R}^N$ with generalised Wentzell (or Wentzell-Robin) boundary conditions $Au + \beta \frac{\partial u}{\partial \nu_A} + \gamma u = 0$ on $\partial\Omega$. These were first introduced by A. D. Wentzell in the context of sub-Markovian semigroups in 1959, but have only been extensively studied in the last ten years. We will discuss several methods commonly used in their study, including form methods and via operator matrices. We will also look at how their spectral and isoperimetric properties are related to those of the Robin problem.

On norms connected with hypergeometric mean

Eder Kikianty, Victoria University

Thursday 5.00 (Poster Session)

Abstract: The nth ($n \geq 2$) Cartesian product of a normed space X can be normed using classical means, exemplified by the (vector-valued analogue of) ℓ^p norm. The hypergeometric mean of n vectors in X is calculated by evaluating a number of different weighted arithmetic means, indexed by the points of an $(n-1)$-simplex, and then finds the $L^p(X)$ norm of this collection of means by integrating over the simplex. For $p \geq 1$, this procedure gives a norm on X^n, and is called the $p-HH$ norm. The $p-HH$ norm preserves more of the geometrical structure of X, in contrast to the ℓ^p-norm, although they are equivalent on X^n. Extending this norm to the space of sequences in X reveals fundamental differences of the $p-HH$ norms and the ℓ^p-norms. Although the resulting sequence spaces all lie between $\ell^1(X)$ and $\ell^\infty(X)$, the resemblance to $\ell^p(X)$ ends there.

This is joint work with Gord Sinnamon.

Multiparameter singular integrals with non-smooth kernels on product domains

Ji Li, Macquarie University

Thursday 5.00 (Poster Session)

Abstract: In this paper, we obtain the L^p ($1 < p < \infty$) estimate and the endpoint estimate, namely $L \log^+ L$ to $L^{1,\infty}$ for a type of singular integrals with non-smooth kernels on product domains, especially the Marcinkiewicz-type spectral multipliers. The main tools we used are to establish the product Hardy and BMO spaces associated with operators satisfying the Davies-Gaffney estimate, and then obtain the duality between Hardy and BMO spaces.

Local Hardy Spaces of Differential Forms

Andrew Morris, The Australian National University

Thursday 5.00 (Poster Session)

Abstract: We define local Hardy spaces of differential forms $h^p_{\mathcal{D}}$ that are adapted to a class of first order differential operators $\mathcal{D}$ on a complete Riemannian manifold M with at most exponential volume growth. If D is the Hodge–Dirac operator on M and $\Delta = D^2$ is the corresponding Laplacian, then the local geometric Riesz transform $D(\Delta + aI)^{-1/2}$ is bounded on h^p_D provided that a is large enough compared to the exponential growth of M. A characterisation of $h^1_{\mathcal{D}}$ in terms of local molecules is also obtained.

This is a local version of the Hardy spaces H^p_D introduced recently by Auscher, McIntosh and Russ. The localisation techniques, exponential off-diagonal estimates and atomic characterisations that we have used will be illustrated. This is joint work with Andrea Carbonaro and Alan McIntosh.

An abstract approach to domain perturbation for parabolic equations

Parinya Sa Ngiamsunthorn, The University of Sydney

Thursday 5.00 (Poster Session)

Abstract: We study the behaviour of solutions to Dirichlet and Neumann boundary value problems under singular perturbation of the domain. Let Ω_n be a sequence of bounded domains in $\mathbb{R}^N$. It is known for elliptic boundary value problems that the sequence of solutions u_n on Ω_n converges strongly to the solution of the limit problem if Ω_n satisfies Mosco convergence of the appropriate function spaces. In this work we investigate the convergence of solutions for the corresponding parabolic equations.

The Regular Part of Sectorial Forms

Manfred Sauter, University of Auckland

Thursday 5.00 (Poster Session)

Abstract: In a recent article W. Arendt and A.F.M. ter Elst associate to a given not necessarily closed sectorial form in a natural way an m-sectorial operator. Thereby they generalize the notion of the regular part of a positive symmetric form due to B. Simon. Motivated by H. Vogt's investigation of the regular part of positive symmetric forms for elliptic differential operators, we study properties of the generalized regular part of sectorial forms. We show that taking the regular part of a sectorial form associated with a sectorial differential operator still amounts to a modification of the coefficients of the form. This is joint work with A.F.M. ter Elst.

Hypersurface L^p Estimates for Approximate Eigenfunctions of a Differential Operator

Melissa Tacy, The Australian National University

Thursday 5.00 (Poster Session)

Abstract: Consider a differential operator P on a compact Riemannian Manifold M. We ask what can be said about the L^p size of the (approximate) eigenfunctions of P when restricted to a hypersurface H. We prove estimates of the form $\|u_j\|_{L^2(H)} \leq C\lambda^{\delta(n,p)}$. The estimates are obtained by converting the eigenfunction problem into a problem concerning norms of restricted evolution operators and then apply Strichartz estimates with equal time and space weighting.