Abstract : Classical Hodge theory deals with projective and smooth algebraic varieties defined over $\mathbb{C}$. If X is such a variety defined over the subfield $\mathbb{Q}$, then associated to X are two cohomology theories :

1) The Betti cohomology of X, denoted $H^*_B(X)$. It is defined as the singular cohomology of the underlying space of the complex points of X, $X(\mathbb{C})$, with coefficients in $\mathbb{Q}$.

2) the algebraic de Rham cohomology of X, denoted by $H^*_{dR}(X)$. This is defined as the hypercohomology of the de Rham complex $\Omega^*_{X/\mathbb{Q}}$.

There is a canonical isomorphism $H^*_{dR}(X) \otimes_{\mathbb{Q}} \mathbb{C} \simeq H^*_B(X) \otimes_{\mathbb{Q}} \mathbb{C}$. If we choose bases for these two $\mathbb{Q}$-vector spaces, this isomorphism is given by an element in $GL_N(\mathbb{C})$, called the period matrix. In general little is known about the entries in this matrix, although the degree of transcendence of the field generated over $\mathbb{Q}$ by its entries, was conjectured by Grothendieck to have dimension equal to that of a reductive algebraic group defined over $\mathbb{Q}$, the Mumford-Tate group associated to X.

We will discuss the p-adic analogues of this classical situation. Beginning with the work of Barsotti, Tate, Grothendieck during the 1950's and 1960's, the theory of p-divisible groups developed. In his Nice ICM talk, Grothendieck raised the problem of the "mysterious functor". In 1978 Fontaine introduced a ring, B_{crys}, and a field, B_{dR}, containing it, each equipped with additional structure, in order to obtain p-adic analogues of the period isomorphism described above. In Fontaine's 1982 Annals paper, a number of conjectures relating p-adic etale cohomology to crystalline and de Rham cohomology were made. These conjectures have all been established due to the work, over the last thirty years of various mathematicians including Fontaine, Bloch, Breuil, Colmez, Faltings, Gabber, Hyodo, Illusie, Kato, Kisin, Messing, Niziol, Olsson, Tsuji, ... Just as with classical Hodge theory, p-adic Hodge theory has had important applications in number theory and arithmetic geometry, including, in particular, the proof, by Khare and Wintenberger , of Serre's Modularity Conjecture.