BEN ANDREWS
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1 'Exotic' differential geometry

The usual (Euclidean) differential gometry of curves or surfaces studies those properties which are invariant under rigid motions (i.e. translations and rotations). In certain circumstances it is also interesting to consider properties which are invariant under other groups – for example, the group of affine transformations or the smaller group of special affine transformations (this produces 'affine differential geometry'); the group of similarities (i.e. totations and translations as well as scalings); conformal (Mobius) transformations on the plane or in space, which gives rise to 'inversive' differential geometry; or the group of projective transformations ('projective differential geometry'). These geometries have some features in common but also some interesting differences.

A project could investigate one or several of the following topics: Variational problems in the setting of such exotic geometries (i.e. finding and solving analogues of the problem of finding a shortest curve between two points, or of finding a surface of least area spanning a given curve); analogues of some famous Euclidean-geometric results in some other geometries (for example, the 4-vertex theorem in Euclidean geometry says that a closed curve in the plane must have at least 4 vertices — points where the curvature attains a maximum or minimum — and a similar 6-vertex theorem should hold in affine differential geometry; and classes of special curves or surfaces (analogous to ruled surfaces, developable surfaces, minimal surfaces, constant curvature surfaces, and so on in Euclidean geometry). These would all involve a mixture of differential geometry and differential equations.

2: Inequalities for fundamental frequencies

Imagine a drum made from a flexible membrane in the shape of a region Ω in the plane. Vibrations in the membrane are governed by the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \Delta u,$$

where $\Delta = \frac{\partial^2}{\partial_x^2} + \frac{\partial^2}{\partial y^2}$, with the condition $u = 0$ on the boundary of Ω. The solution u can be expanded in the form

$$u(x,t) = \sum_{k=1}^{\infty} a_k \varphi_k(x) e^{i\lambda_k t}$$

for some constants a_k, where the functions $\varphi_k : \Omega \to \mathbb{R}$ satisfy the eigenvalue equation

$$\Delta\varphi_k + \lambda_k^2 \varphi_k = 0$$

with $\varphi_k = 0$ on the boundary of Ω. The numbers λ_k are the eigenvalues or fundamental frequencies of the domain Ω. Assume that $0 < \lambda_1 \le \lambda_2 \le \lambda_3 \ldots$.

There are many things known, and many more conjectured, about the eigenvalues of domains in the plane – for example, it is known that the round disk has smaller λ_1 than any other domain of the same area, and that $\lambda_1(\Omega_1) \geq \lambda_1(\Omega_2)$ whenever $\Omega_1 \subset \Omega_2$.

This project would survey the known results and conjectures in this area, and attempt to make some generalisations.